Summer Prep Work for Students Entering Geometry

Operations, Expressions, and Equations

1. Evaluate when \( a = 2, \ b = 0.5, \ c = 3, \ d = \frac{3}{4} \)

\[(cd)^3 + ab\]

2. The expression \( x(x + 3) \) is the same as:

a.) \( 2x + 3 \)  

b.) \( x^2 + 3 \)  

c.) \( 2x + 3x \)  

d.) \( x^2 + 3x \)

3. You purchase notebooks for \$0.75\) each and pens for \$0.99\) each. Write an algebraic expression for the total you spend on notebooks and pens.

The chart below shows the monthly budgets (in dollars) of 5 people for 6 categories of expenses. Use the chart to answer questions 4-6

<table>
<thead>
<tr>
<th>CATEGORY</th>
<th>JIM</th>
<th>JOAN</th>
<th>JACK</th>
<th>JULIE</th>
<th>KATHY</th>
</tr>
</thead>
<tbody>
<tr>
<td>RENT</td>
<td>250</td>
<td>375</td>
<td>300</td>
<td>185</td>
<td>205</td>
</tr>
<tr>
<td>AUTO</td>
<td>150</td>
<td>275</td>
<td>180</td>
<td>320</td>
<td>250</td>
</tr>
<tr>
<td>FOOD</td>
<td>95</td>
<td>130</td>
<td>100</td>
<td>150</td>
<td>110</td>
</tr>
<tr>
<td>CLOTHES</td>
<td>75</td>
<td>50</td>
<td>110</td>
<td>80</td>
<td>175</td>
</tr>
<tr>
<td>RECREATION</td>
<td>20</td>
<td>30</td>
<td>40</td>
<td>15</td>
<td>50</td>
</tr>
<tr>
<td>SAVINGS</td>
<td>50</td>
<td>150</td>
<td>75</td>
<td>90</td>
<td>115</td>
</tr>
<tr>
<td>TOTAL</td>
<td>640</td>
<td>1010</td>
<td>805</td>
<td>840</td>
<td>905</td>
</tr>
</tbody>
</table>

4. What percent of Jim’s total budget goes for rent?

5. What is the average (mean) clothing budget for the 5 people?

6. What is the median amount budgeted for rent in the budgets of the 5 people?

7. Simplify: \( \frac{1}{4} \cdot \frac{2}{5} \)
8. Evaluate: \( 7 \div \frac{-2}{5} \)

9. Mrs. Barlowski made butterscotch pudding using \( \frac{1}{4} \) of a cup of water, \( \frac{3}{8} \) a cup of whipping cream, \( \frac{2}{3} \) a cup of nuts, and 2\% cups of milk. How many cups of ingredients did Mrs. Barlowski use in total?

10. Two-thirds of a number is eight-fifths. What is the number?

11. Solve for \( x \): \( 3(x + 5) = 8x + 18 \)

12. Solve for \( t \): \( \frac{t - 7}{4} = 11 \)

13. Solve: \( 9x - 18 = 6x - 8 + x \)

14. Solve: \( \frac{3}{7} = \frac{x - 2}{21} \)

15. Solve for \( y \): \( 5y - 3 = 3x + y \)

16. Rewrite in function form (\( y = mx + b \)).
   
   \[ 3x - 2y = 10 \]
17. Solve for \( n \).

\[
\frac{3n}{4} = w
\]

The formula for finding degrees Celsius from degrees Fahrenheit is \( C = \frac{5}{9}(F - 32) \).

Use this information for questions 17 and 18.

18. Solve the equation for \( F \).

19. If the temperature is currently 25 °Celsius, how many degrees Fahrenheit is this?

20. A store has 49 cartons of yogurt, some plain, some with blueberry flavoring. There are six times as many cartons of plain yogurt as blueberry. How many cartons of plain yogurt are there?

**Percents and Proportions**

21. Ben was working for $8.00 an hour but was given a 20% pay increase. What is his new hourly rate? Grid in your answer.

22. If you can travel 320 miles on 12 gallons of gas, how much gas would you need to travel 715 miles? Round your answer to the nearest whole gallon.


**Linear Equations**

24. Find the \( x \)- and \( y \)-intercepts of the equation \( 3x - 6y = 24 \). Be sure to list your answers as ordered pairs.

25. Write the following slope-intercept equation in standard form (\( ax+by=c \)):  
   \[ y = 7x - 1 \]

26. What is the slope and \( y \)-intercept in the equation:  
   \[ y = \frac{3}{4}x - 4 \]

27. Find the slope and \( y \)-intercept of the equation  \( 8x - 4y = -64 \)

28. Write the slope-intercept form (\( y=mx+b \)) of the equation for the line below.

29. Write an equation of the line passing through \( (4, 4) \) and \( (2, 1) \).

   a. \( y = -\frac{2}{3}x + 1 \)
   
   b. \( y = \frac{2}{3}x - \frac{1}{3} \)
   
   c. \( y = \frac{3}{2}x - 2 \)
   
   d. \( y = -\frac{3}{2}x + 2 \)

30. Find the equation of the line that passes through the pair of points. You may use any method you prefer. \( (−3, 2) \), \( (−5, 6) \)
31. Acme Rental Car charges $20 base fee plus 11 cents per mile to rent a certain car. TNT Car Rentals charges $24 base fee plus 6 cents per mile to rent the same car.
   a. Write an equation for each car agency to represent the cost as a function of the number of miles driven.

   b. Find the number of miles for which the costs are the same. Round your answer to the nearest tenth of a mile.

**Simplifying expressions & Properties**

32. Simplify \(-\frac{5x^{-3}yz}{30xy^{-2}z}\)

33. Simplify the expression \((2q^7)(7q)(6q^9)\)

34. Expand the product by distributing: \(2n(n^2 + 3n + 4)\)

35. Expand the product: \((x - 3)(x^2 + x - 1)\)

36. Find the value of \(x\) by using the quadratic formula: \(x^2 - x - 6 = 0\)

37. Use the distributive property: \(5xy(xy^2 + 6y)\)

38. Simplify: \((b^2)(b)(b^4)\).

39. Multiply: \(5x(x + y)\).

40. Simplify: \((3x^2 + 7x - 4) + (2x^2 - 3x - 9)\).
41. Solve by using the quadratic formula:  \[ x^2 + 2x - 48 = 0 \]

42. Solve for \( x \). Provide both answers in simplified radical form and estimated to the nearest hundredths.  \( x^2 = 32 \)

**Inequalities**

43. Find a compound inequality that the graph could represent.

\[ \text{[A]} \ -6 \leq x < 7 \quad \text{[B]} \ x < -6 \text{ or } x \geq 7 \quad \text{[C]} \ x \leq -6 \text{ or } x > 7 \quad \text{[D]} \ -6 < x \leq 7 \]

44. Graph the linear inequality  \( y < 5x - 5 \) then list at least 3 ordered pairs that represent solutions (satisfy the inequality).

45. During the summer, you want to earn at least $300 per week. You earn $12 per hour working as a lifeguard, and you earn $10 per hour working at a retail store.

   a. Write an inequality statement for the problem above.

   b. What are the minimum number of hours you need to lifeguard in order to achieve your weekly financial goal, assuming your retail work hours are 20 hours per week?
**Mixed Word Problems**

46. John has earned 312 out of 400 possible points (78%). He wants to raise his average to at least a B− (80%). Is this possible if there are 150 points left in the marking period? What are the fewest points needed to reach his goal of 80%. Show your work.

47. Lizette and Courtney enter a partnership. Lizette puts in $5,000 and Courtney puts in $4,000. Their profit is $1,200 after the first year. They will split the profit proportional to their contributions. How much does each get?

48. A coat you for which you paid $75 had been marked down 40%. What was the original cost of the coat?

49. Lincoln High School has 623 students. The ratio of girls to boys is approximately 2 girls to 3 boys. How many girls are enrolled in Lincoln Middle School? How many boys?

50. Rachel started making homemade cards to send to friends and family and to sell at the local craft fair. The scatter plot shows how many cards Rachel made each hour she worked. Predict the total number of cards Rachel can make in hour 24 by drawing a line of best fit and finding its equation.
51. On an upcoming field trip to the math and science museum, a bus will cost $450 for the trip and the admission to the exhibit is $7.00 per person.

   a. Create an equation (function) that would determine the total cost for any number of students going on the trip (you may assume the bus will hold any number of students).

   b. If the budget for the trip is $925, how many students would be able to go?

   c. How much would each student pay to cover the entire cost of the trip if 58 students go on the trip?

52. A regulation Little League baseball field is a square with bases exactly 60 feet apart. Use the Pythagorean Theorem to determine the distance from home plate to 2nd base on a Little League field.

53. A bag contains 15 red marbles, 10 white marbles, 25 blue marbles and 10 green marbles.

   a. If Fred Baxter takes a marble at random from the bag, what is the probability that it is green?

   b. If Fred gets a red marble, and doesn’t return it to the bag, what is the probability that Sgt. O’Malley will pick a white marble?

54. John has a part time job. His pay varies directly with the hours worked. If his wages are $89.90 for working 5 ½ hours, how much will he be paid for working 30 hours?
55. Sue’s bank charges $2.25 a month plus $0.10 per check. How many checks did she write if her bank fees were $3.65 last month?

56. Ray has scores of 75, 82, 94, and 77 on his first four science tests. What must he score on the next test to have an average of at least 85?

57. Explain the difference between a line with a zero slope and a line with an undefined slope. Explain the difference between positive slope and negative slope.

Sketch and label examples of each.

Zero slope

Undefined slope

Positive slope

Negative slope

58. Solve the system of equations by using the elimination method.

\[ 2x + y = 7 \]
\[ x + y = 1 \]

59. You purchase a used car for $13,500. It decreases in value by 15% per year. Using \[ y = a(1 - r)^t \], find the remaining value of the car in 4 years.
60. Jay Walker had a starting salary of $50,000 for his job at city hall. Each year, his salary increased by 4%. What will his salary be after 6 years on the job?

61. Saul Otswood paid $5,000 for a new lathe for his carpentry business. This particular machine depreciates (decreases in value) at a rate of 9.5% per year. After how many years will the lathe be worth less than half of its original value?

62. Saul Otswood invested $4,000 in an account that paid 2.3% annual interest compounded weekly in hopes of being able to pay $5,000 for his next new lathe. How much is in the account after 10 years?

63. Graph and solve for x: \[2 \cdot 3^{x} = 120\]

64. Farmer Fenster has two identical square fields with a combined area of 32 square kilometers. He wants to put up a fence on one side of one of the fields. How long a fence does he need?
65. Kevin is a human cannonball at a circus and is launched into the air with an initial velocity of 150 ft/sec from a height of 5 feet. The relationship between height, \( h \), in feet and time, \( t \), in seconds can be modeled by the function
\[
h(t) = -16t^2 + 150t + 5.
\]

a. Find the maximum height (to the nearest foot) that Kevin reaches. Also, find out how much time (to the nearest tenth of a second) elapses before Kevin reaches his maximum height.

Kevin reaches a maximum height of ________ feet after ________ seconds.

b. Determine the amount of time (to the nearest tenth of a second) it takes for Kevin to reach a net that is 12 feet above the ground.

It takes him ________ seconds to reach the net.

66. Using the figure to the right:

a. Write a polynomial representing the shaded area (the area inside the rectangle and outside the center triangle).

b. Find the value of \( x \), given the total shaded area is 50 square inches.

67. The triangle below has a perimeter of 20 cm. How long is each side of the triangle?
68. A photocopier is used to make a similar, reduced copy of a document. The original document is 24 centimeters wide and 33 centimeters tall. The copy is 15 centimeters wide. What is the height of the copy, to the nearest tenth of a centimeter?

- a. 24.0 cm
- b. 20.6 cm
- c. 21.6 cm
- d. 52.8 cm

69. A soccer player makes a corner kick to another player, as shown. To the nearest yard, how far does the player kick the ball?

- A) 7 yards
- B) 38 yards
- C) 42 yards
- D) 52 yards

70. Find the distance between Player A and Player B. Each square on the grid is one square foot.
Formulas:

Area of Triangle:  \( A = \frac{1}{2}bh \)

Area of Circle:  \( A = \pi r^2 \)

Circumference:  \( C = \pi d \)

Pythagorean Theorem:  \( a^2 + b^2 = c^2 \)

Standard Form of a Quadratic:  \( ax^2 + bx + c = 0 \)

Quadratic formula:  
\[
    x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}
\]

Quadratic Axis of Symmetry Formula:  
\[
    x = -\frac{b}{2a}
\]

Exponential Growth and Decay

\[
\begin{align*}
    A &= P(1 + r)^t \\
    A &= P(1 - r)^t \\
    A &= P\left(1 + \frac{r}{n}\right)^{nt} \\
    A &= Pe^{rt}
\end{align*}
\]
Solutions

1. 65
2. D.
3. \(.75n + .99p = T\)
4. 39%
5. $98
6. 250
7. \(\frac{5}{8}\)
8. \(\frac{-35}{2}\)
9. \(\frac{11}{12}\)
10. \(x = 2.4\)
11. \(x = \frac{-3}{5}\)
12. \(t = 51\)
13. \(x = 5\)
14. \(x = 11\)
15. \(y = \frac{3}{4}x + \frac{3}{4}\)
16. \(y = \frac{3}{2}x - 5\)
17. \(n = \frac{4w}{3}\)
18. \(F = \frac{9}{5}C + 32\)
19. 77
20. \(b = 7\) \(p = 42\)
21. $9.60
22. 27 gallons
23. $450
24. \((0, -4) (8, 0)\)
25. \(-7x + y = -1\)
26. \(m = \frac{3}{4}\) \(b = -4\)
27. \(m = 2\) \(b = 16\)
28. \(y = 2x + 5\)
29. C.
30. \(y = -2x - 4\)
31. a. \(A = .11m + 20\)
   b. 80 miles
32. \(\frac{-y^3}{6x^4}\)
33. 84q^{17}
34. \(2n^3 + 6n^2 + 8n\)
35. \(x^3 - 2x^2 - 4x + 3\)
36. \(x = 3, x = -2\)
37. \(5x^2y^3 + 30xy^2\)
38. \(b^7\)
39. \(5x^2 + 5xy\)
40. \(5x^2 + 4x - 13\)
41. \(x = -8, x = 6\)
42. \(4\sqrt{2} or 5.66\)
43. D
44. 
45. a. \(12L + 10R \geq 300\)
   b. at least 8.3 hours
46. 40 more points
47. $666.67 and $533.33
48. $125
49. 250 girls, 373 boys
50. about 60 cards
51. a. \(y = 7x + 450\)
   b. \(x = 67\)
   c. $14.76
52. 84.85 ft.
53. a. \(\frac{1}{6}\)  b. 17%
54. $490.36
55. \(x = 14\)
56. \(x = 97\)
57. zero slope is horizontal (no rise)
   Undefined slope is vertical (no run)
58. \(x = 6, y = -5\)
59. $7,047.08
60. $63,265.95
61. \(t \approx 7\) years
62. $5,034.14
63. \(x \approx 3.73\)
64. 4 km
65. a. 356.6 feet after 4.69 seconds
   b. 9.33 seconds

66. a. \( A = \frac{1}{2} (x + 6)^2 \)   b. \( x = 4 \)

67. \( x = 5 \)

68. B.
69. C.
70. \( 2\sqrt{5} = 4.47 \text{ ft} \)